THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY IN NEW YORK.

THE one hundred and eighty-fourth regular meeting of the Society was held in New York City on Saturday, April 29, 1916. The attendance at the morning and afternoon sessions included the following fifty-one members:

Dr. J. W. Alexander, II, Dr. F. W. Beal, Mr. D. R. Belcher, Dr. A. A. Bennett, Professor E. W. Brown, Professor B. H. Camp, Professor C. W. Cobb, Dr. Emily Coddington, Professor F. N. Cole, Professor J. L. Coolidge, Dr. Lennie P. Copeland, Professor Elizabeth B. Cowley, Professor Louise D. Cummings, Mr. C. H. Currier, Dr. H. B. Curtis, Professor L. P. Eisenhart, Professor H. B. Fine, Dr. C. A. Fischer, Professor C. C. Grove, Dr. Olive C. Hazlett, Professor E. R. Hedrick, Professor E. V. Huntington, Professor Dunham Jackson, Mr. Glenn James, Mr. S. A. Joffe, Professor Edward Kasner, Dr. L. M. Kells, Professor C. J. Keyser, Mr. Harry Langman, Dr. P. H. Linehan, Professor James Maclay, Dr. R. L. Moore, Professor Frank Morley, Mr. G. W. Mullins, Mr. George Paaswell, Professor James Pierpont, Professor H. W. Reddick, Professor R. G. D. Richardson, Mr. J. F. Ritt, Dr. Caroline E. Seely, Dr. H. M. Sheffer, Professor Clara E. Smith, Professor D. E. Smith, Professor P. F. Smith, Professor Oswald Veblen, Mr. J. H. Weaver, Mr. H. E. Webb, Dr. Mary E. Wells, Professor H. S. White, Mr. J. K. Whittemore, Dr. C. E. Wilder.

The President of the Society, Professor E. W. Brown, occupied the chair, being relieved by Vice-President E. R. Hedrick. The Council announced the election of the following persons to membership in the Society: Dr. E. T. Bell, University of Washington; Professor T. R. Eagles, Howard College; Mr. Glenn James, Purdue University; Dr. J. O. Hassler, Chicago, Ill.; Professor G. N. Watson, University College, London; Mr. J. H. Weaver, West Chester, Pa. Six applications for membership in the Society were received.

Professor D. R. Curtiss was reelected a member of the Editorial Committee of the *Transactions*, to serve for three years beginning October 1, 1916. The resignation of Professor

L. E. Dickson as member of the Editorial Committee was accepted to take effect on October 1, 1916, and Professor L. P. Eisenhart was appointed to fill out the remaining year of Professor Dickson's term. Committees were appointed to prepare a list of nominations of officers and other members of the Council to be elected at the annual meeting, and to consider the matter of the publication of the Harvard Colloquium Lectures. A committee was also appointed, consisting of Professors E. V. Huntington, E. B. Wilson, E. H. Moore, R. C. Archibald, and T. H. Gronwall, to consider in cooperation with other scientific bodies the question of the classification of technical literature.

Thirty-seven members and friends assembled at the dinner after the meeting.

The year 1916 marks the twenty-fifth anniversary of the broadening out of the Society into a national organization, and of the founding of the BULLETIN. It is proposed to celebrate this anniversary in an appropriate manner at the coming summer meeting. Some seventy-five of those who joined the Society in or prior to 1891 have retained their membership during these twenty-five years. It is hoped that a large number of these older members may be present at the summer meeting and that a large representation of the younger generation may also be present to take over the responsibility for the Society's progress during the next quarter century.

The following papers were read at this meeting:

(1) Dr. SAMUEL BEATTY: "Derivation of the complementary theorem from the Riemann-Roch theorem."

(2) Mr. J. F. RITT: "The resolution into partial fractions of the reciprocal of an entire function of genus zero."

(3) Mr. J. F. RITT: "Linear differential equations of infinite order with constant coefficients."

(4) Professor C. J. KEYSER: "Concerning autonomous doctrines and doctrinal functions."

(5) Professor EDWARD KASNER: "Element transformations of space for which normal congruences of curves are invariant."

(6) Mr. J. H. WEAVER: "Some extensions of the work of Pappus and Steiner on tangent circles."

(7) Professor J. L. COOLIDGE: "New definitions for Plücker's numbers."

(8) Professor G. C. EVANS: "Integral equations whose

kernels satisfy a certain difference equation in variable differences."

(9) Professor DUNHAM JACKSON: "An elementary boundary value problem."

(10) Professor L. P. EISENHART: "Transformations of conjugate systems with equal point invariants."

(11) Dr. A. A. BENNETT: "An existence theorem for the solution of a type of real mixed difference equation."

(12) Dr. A. A. BENNETT: "A case of iteration in several variables."

(13) Mr. R. W. BRINK: "Some integral tests for the convergence and divergence of infinite series."

(14) Mr. GLENN JAMES: "A theorem on the non-summability of a certain class of series."

(15) Dr. F. J. MCMACKIN: "Some theorems in the theory of summable divergent series."

(16) Mr. J. R. KLINE: "A definition of sense on plane curves in non-metrical analysis situs."

(17) Professor H. B. FINE: "On approximations to a solution of a system of numerical equations."

(18) Professor B. H. CAMP: "Fourier multiple integrals."

(19) Dr. G. A. PFEIFFER: "On the conformal mapping of curvilinear angles."

(20) Professor G. C. EVANS: "Proof of Green's theorem by approximating polynomials."

(21) Dr. A. R. SCHWEITZER: "On a type of quasi-transitive functional equations."

(22) Dr. J. W. ALEXANDER, II: "Some generalizations of the Jordan theorem."

(23) Dr. C. E. WILDER: "Expansion problems of ordinary linear differential equations with auxiliary conditions at several points."

(24) Professor E. V. HUNTINGTON: "A simple example of the failure of Duhamel's theorem."

(25) Professor W. F. OSGOOD: "Note on functions of several complex variables."

(26) Professor W. A. WILSON: "On separated sets."

Mr. Brink's paper was communicated to the Society by Professor Birkhoff; Dr. McMackin was introduced by Professor Keyser, and Mr. Kline by Dr. R. L. Moore. The papers of Dr. Beatty, Professor Evans, Mr. Brink, Dr. Pfeiffer, Dr. Schweitzer, Dr. Alexander, Professor Osgood, and Professor Wilson were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. By making use of the idea of complementary bases, without which the complementary theorem cannot be even stated, Dr. Beatty derives the complementary theorem from the less general Riemann-Roch theorem.

2. In Mr. Ritt's first paper, which is auxiliary to the second, are discussed the conditions under which the reciprocal of an entire function $G(z)$, of genus zero, can be resolved into simple elements as if $G(z)$ were a polynomial. If the zeros of $G(z)$ are $a_1, a_2, \cdots, a_n, \cdots$, the resolution is possible when, on and after a certain point,

$$\left|\frac{a_{n+1}}{a_n}\right| > 1 + \frac{k}{n},$$

where $k > 2$.

3. The first part of Mr. Ritt's second paper develops the theory of the operator

$$A = \left(1 - \frac{D}{a_1}\right)\left(1 - \frac{D}{a_2}\right) \cdots \left(1 - \frac{D}{a_n}\right) \cdots,$$

where D denotes differentiation and where

$$\sum_{n=1}^{\infty} \frac{1}{|a_n|}$$

is convergent. This operator, which, strangely enough, does not seem to have been discussed before, has the entire space of analytic functions for its domain of applicability. In the second part is discussed the most general solution of the equation $A\varphi(z) = 0$. The solutions are shown to be uniform and to have no isolated singularities. The analytical representation of the solutions is discussed. There is obtained, incidentally, a known result in the theory of analytic prolongation. In the third part of the paper, which will furnish material for future investigations, more general equations are considered, the principal object being to apply the results to the above equation $A\varphi(z) = 0$.

4. The undefined or primitive terms or elements in a postulationally established theory admit of infinitely many interpretations. Hence such terms are variables. Hence the postulates or primitive so-called propositions are not propositions, being neither true nor false prior to specific interpretation, but are propositional functions, and the same holds of the deduced so-called propositions. Accordingly, the theory constituted by such propositional functions, being neither true nor false prior to assignment of specific values to the primitive variables involved, is not a doctrine but is, Professor Keyser contends, something that may be fitly called a doctrinal function. Just as a propositional function is a source of innumerable propositions, some true, some false, the true ones being called values of the propositional function, so a doctrinal function is a source of innumerable doctrines, some true, some false, arising from diverse specific interpretations of the primitive variables. The true doctrines thus arising ought, naturally, to be called the values of the doctrinal function. It thus appears that Hilbert's so-called "Foundations of Geometry," for example, is, strictly speaking, not a geometry nor any other specific doctrine. It is a 3-dimensional euclidean doctrinal function, of which ordinary euclidean geometry is one value, viz., that value that arises from assigning to the primitive variables (Hilbert's "point," "line," and "plane") such meanings as Hilbert did not assign but Euclid did assign (unfortunately under the caption of "definitions" instead of descriptions). No system of primitive propositional functions, or postulates, is satisfied by a set of specific entities that do not have "excessive meaning," i. e., meaning over and above that required merely to satisfy the system. The various doctrines arising from a given doctrinal function are thus known and distinguished by the differing excessive meanings of the entities involved. For a doctrine to be geometric, part of the excessive meaning of the primitive entities must be the concept of spatial extension.

5. Professor Kasner determines all transformations of lineal elements (x, y, z, y', z') of space such that every normal congruence of curves shall be converted into a normal congruence. The infinite group obtained is isomorphic with the group of contact transformations in space. The only transformations in the new group which convert curves into curves

are the conformal transformations, which form a 10-parameter subgroup. The results are of interest in connection with the optics of general isotropic media.

6. Pappus in Book IV of the Collection develops some properties of infinite systems of tangent circles A which are tangent to two given circles. Mr. Weaver's paper extends these results in two directions.

(1) He finds the analytic expression for the radius of the nth circle in the infinite series, and also the expression for certain other allied lines, and from these expressions develops some infinite series which may be summed geometrically. These series closely resemble those given by Fabry,* from an analytic point of view.

(2) He develops a projective method for constructing conics and investigates some projective properties of tangents and normals to the conics determined by the given circle and the series A. The results here are supplementary to those of Steiner on the same problem,† and include additional extensions in the light of modern projective theory.

7. Professor Coolidge's paper gives new definitions for the order, class, and deficiency of an algebraic plane curve suitable to the case where the curve is required to be real.

8. Professor Evans's first paper appears in full in the present number of the BULLETIN.

9. Professor Jackson's paper appeared in full in the May BULLETIN.

10. Professor Eisenhart considers pairs of surfaces S and S_1, so related that for the congruence of the joins of corresponding points M and M_1, the developables meet S and S_1 in conjugate systems of curves. Let S^{-1} and S' denote the Laplace transforms of S with respect to these conjugate curves, taken as parametric. The tangent planes to S osculate one family of parametric curves on S^{-1} and the other family on S'. In each of these osculating planes there is a pencil of conics tangent to the two curves at the points of osculation. These

* Théorie des Séries à Termes constants, Chap. IV.

† Steiner, Werke, herausgegeben von Weierstrass, vol. I, pp. 47 and ff.

conics determine involutions on the line of intersection of the tangent planes to S and S_1. Involutions are likewise determined on these lines by similar pencils of conics in the tangent planes to S_1. These involutions are the same only in case the parametric conjugate systems on S and S_1 have equal point invariants, in which case S and S_1 are in the relation of a transformation K, previously considered by the author. Certain pencils of quadrics of singular interest may be associated with these pencils of conics. The interrelations of these various configurations are determined. The paper will appear in the *Annals of Mathematics*.

11. In this paper Dr. Bennett points out the fact that a real mixed difference equation of the form

$$y_{h+1}^{(0)} = F[x;\ y_0^{(0)},\ y_0^{(1)},\ \cdots,\ y_0^{(k)};$$
$$y_1^{(0)},\ \cdots,\ y_1^{(k_1)};\ \cdots;\ y_h^{(0)},\ \cdots,\ y_h^{(k_h)}],$$

where by $y_i^{(j)}$ is meant $d^j y(x+i)/dx^j$, will under certain simple and general restrictions have as solutions only functions which are continuous together with all of their derivatives for all non-negative values of x. It is then shown that not only are the independent initial conditions infinite in number, but that solutions may be readily constructed having an infinite number of degrees of freedom, where the Taylor's series of the desired solution is assigned arbitrarily, as divergent series if desired, at each of the points $x = i,\ i = 0, 1, \cdots, h$. Use is made of some of the results obtained by Mr. J. F. Ritt in a forthcoming article in the *Annals of Mathematics*. The present paper, also, will appear in the *Annals*.

12. Dr. Bennett here examines a special case of iteration in several variables. The essential equivalence is pointed out between difference equations in several variables and those functional equations which constitute the subject matter of iteration. In particular, the author examines equations of the form $F_{(i)}[u_1, u_2, \cdots, u_n] \equiv au_i + ((u^2)),\ i = 1, 2, \cdots, m$, where by $((u^2))$ is meant a power series in the m variables $(u_1, u_2, \cdots, u_m)$ for which no term appears of lower than the second degree in the set of m variables together. For the case in which $|a| \neq 0, \neq 1$, an explicit solution is obtained in terms of the integral iterates by a formula which is obtained

from an extension of Newton's interpolation formula. The limiting forms are also discussed. This paper constitutes a partial extension of the general theory of one variable contained in a previous paper by the author, *Annals of Mathematics*, volume 17 (1915). This paper will also appear in the *Annals*.

13. The recurrence formula that defines the general term of an infinite series may be regarded as a difference equation in the partial sum of the series. In Mr. Brink's paper methods are given whereby such a difference equation may be replaced by a differential equation whose solution behaves for large integral values of the variable like the partial sum of the series. In this way the author obtains a sequence of integral tests for the convergence and divergence of series. The Maclaurin-Cauchy test is given with extended conditions as the first of this sequence. Another of the simpler tests presented makes use of the function $r(x)$ which has the property that $r(n)$ is the ratio of u_{n+1} to u_n, u_n being the general term of the series. Under certain restrictions upon $r(x)$ the series converges or diverges with the integral

$$\int_a^\infty e^{\int_a^x \log r(x)dx}\, dx.$$

By means of this and other tests of the sequence a large number of the classical convergence tests are easily established. The tests are readily generalized for multiple series.

14. Mr. James extends the notion of proper divergence to include a certain class of oscillating series. This class contains all series such that for any positive (or negative) C and every M there exists an $m \geqq M$ such that

$$\sum_{p=1}^{p=n} (S_{m+p} - C) \geqq 0, \text{ (or } \leqq 0), n \geqq m+1.$$

A theorem is established from which it follows that the methods of Borel, Cesàro, LeRoy, Cesàro-Riesz, and others cannot sum series of this class.

15. Hardy and Smail showed that the sufficient condition for the continuity of the sum function of a summable divergent

series of continuous functions is the uniform summability of the series, that is, the uniform convergence of the auxiliary limit, as Borel's integral, Cesàro's mean value, etc. Dr. McMackin shows that this is not a necessary condition for the continuity of the sum, and derives conditions which are both necessary and sufficient. For series which oscillate between finite limits quasi-uniform convergence of the auxiliary limit is shown to be both necessary and sufficient in the case of Borel's integral sum, while in the other case it holds for all summable series with continuous terms. It is also shown that a quasi-uniformly convergent series is quasi-uniformly summable according to the definition of quasi-uniform summability given.

16. Mr. Kline proposes the following definition for sameness of sense on two simple closed curves: The sense $A_1B_1C_1$ on the simple closed curve J_1 and the sense $A_2B_2C_2$ on the simple closed curve J_2 are said to be the same with respect to their common exterior E_{12}, if there exist a simple closed curve J_3 lying entirely in E_{12} and three points A_3, B_3, C_3 on J_3 such that (1) if it is impossible to join A_1 to A_3, B_1 to B_3 and C_1 to C_3 by simple continuous arcs which except for their end points lie entirely in E_{13} (the common exterior of J_1 and J_3) and have no points in common, then it is also impossible to join A_2 to A_3, B_2 to B_3 and C_2 to C_3 by simple continuous arcs lying except for their end points entirely in E_{23} (the common exterior of J_2 and J_3) and having no points in common, or (2) if it is possible to join A_1 to A_3, B_1 to B_3, and C_1 to C_3 in the manner above indicated, then it is also possible to join A_2 to A_3, B_2 to B_3 and C_2 to C_3 as indicated.

It is established that if for one choice of the curve J_3 and of three points A_3, B_3, and C_3 lying thereon, the senses $A_1B_1C_1$ and $A_2B_2C_2$ are the same with respect to E_{12}, then these senses are also the same with respect to any other such choice of J_3 and $A_3B_3C_3$.

It is also shown that if the senses $A_1B_1C_1$ on J_1 and $A_2B_2C_2$ on J_2 are opposite with respect to their common exterior and the senses $A_2B_2C_2$ on J_2 and $A_3B_3C_3$ on J_3 are opposite with respect to their common exterior, then the senses $A_1B_1C_1$ on J_1 and $A_3B_3C_3$ on J_3 are the same with respect to their common exterior.

17. Professor Fine's paper is concerned with the proof of the following theorem: Let $f_i(x_1, x_2, \cdots, x_n)$, $i = 1, 2, \cdots, n$, be real functions of the real variables $x_1, x_2, \cdots, x_n$ which have continuous first and second derivatives in the region under consideration. Let $(x_1^0, x_2^0, \cdots, x_n^0)$ belong to this region and let $\xi_1, \xi_2, \cdots, \xi_n$ be the numbers defined by the equations

$$f_i(x_1^0, x_2^0, \cdots, x_n^0) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^0} \xi_k = 0 \quad (i = 1, 2, \cdots, n).$$

Again let S denote the circle, sphere, or hypersphere whose center is $(x_1^0 + \xi_1, x_2^0 + \xi_2, \cdots, x_n^0 + \xi_n)$ and whose radius is $\rho = [\Sigma \xi_k^2]^{\frac{1}{2}}$, and suppose that, for this region S, $D > 0$ denotes the lower bound of the absolute values of the functional determinant of the functions f_i, $M < \infty$ the upper bound of the absolute values of the first minors of this determinant, and $N < \infty$ the upper bound of the absolute values of the second derivatives of the functions f_i.

Then if

$$\left[\sum_{i=1}^{n} f_i^2(x_1^0, x_2^0, \cdots, x_n^0)\right]^{\frac{1}{2}} < \frac{D^2}{n^3 \sqrt{n} M^2 N},$$

the system of equations $f_i(x_1, x_2, \cdots, x_n) = 0$ has one and but one solution in S, and this solution may be approximated to uninterruptedly by successive determinations of $\xi_1^{(j)}, \xi_2^{(j)}, \cdots, \xi_n^{(j)}$ and $x_1^{(j+1)}, x_2^{(j+1)}, \cdots, x_n^{(j+1)}$ by the formulas

$$f_i(x_1^{(j)}, x_2^{(j)}, \cdots, x_n^{(j)}) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^{(j)}} \xi_k^{(j)} = 0,$$

$$x_i^{(j+1)} = x_i^{(j)} + \xi_i^{(j)},$$

the solution being $\lim_{j \to \infty} (x_1^{(j)}, x_2^{(j)}, \cdots, x_n^{(j)})$.

18. By using G. H. Hardy's definition of a non-absolutely convergent multiple integral it is possible to show that an arbitrary function may be expressed as a Fourier double integral under circumstances much more general than is possible when the iterated integral is used. The results are useful in connection with the Fourier quadruple integral which appears in three dimensional physical problems. Professor Camp's paper also considers the continuity, differentiability,

and integrability with respect to parameters of multiple integrals over infinite fields.

19. In a previous paper Dr. Pfeiffer has shown that divergent power series may be formally obtained in seeking a conformal transformation which maps a curvilinear angle upon a rectilinear angle of the same magnitude and which is analytic about the vertex of the angle. In the present paper he shows that there exist functions $f(z)$ which map the interior of the curvilinear angle upon the interior of a rectilinear angle of the same magnitude conformally such that the sum of the first n terms of the power series already referred to represents the function $f(z)$ asymptotically up to order n. The mapping defined by the function $f(z)$ is conformal on the sides of the angle, except of course at the vertex, where it is continuous.

The above refers to angles whose magnitudes are not commensurable with π, for in the contrary case the writer has already shown that all transformations (analytic about the vertex) obtained formally are convergent.

20. Many problems in mathematical physics are expressed in terms of integral relations, and it is only by passing to a limit that the differential equations are secured. There may be nothing in the physical problem which corresponds to the limit. An analysis is therefore desirable which involves rather the integral than the differential relations. Several proofs have been given of Green's theorem on this basis. Professor Evans notes a new proof, possible by means of an expansion in polynomials, of the same nature as the proof of a theorem in de la Vallée Poussin's Cours d'Analyse, volume II, page 24, edition of 1912.

21. Let $\lambda_i(x)$ $(i = 0, 1, 2, \cdots, n+1)$ be functions of a single variable; then Dr. Schweitzer defines the type of quasi-transitive functional equations

$$f\{\lambda_1 u_1, \lambda_2 u_2, \cdots, \lambda_{n+1} u_{n+1}\} = \lambda_0 f\{\phi_1, \phi_2, \cdots, \phi_{n+1}\},$$

where $u_j = f(t_1, t_2, \cdots, t_n, x_j)$, $j = 1, 2, \cdots, n+1$, and ϕ_j denote functions of the variables $x_1, x_2, \cdots, x_{n+1}$. In the present paper the special case $\phi_j = \sum_{k=1}^{n+1} m_{jk} x_k$ only is considered. Then the resulting class of functional equations may be put

into (1, 1) correspondence with the class or linear homogeneous substitutions and one readily defines, at least formally, a representation of an arbitrary finite group of such substitutions. On the other hand, if $\lambda_i(x) = x$ $(i = 0, 1, 2, \cdots, n+1)$ and if $f\{x_1 + y_1, \cdots, x_{n+1} + y_{n+1}\} = f(x_1, \cdots, x_{n+1}) + f(y_1, \cdots, y_{n+1})$, i. e., if $f(x_1, x_2, \cdots, x_{n+1}) = \sum_{j=1}^{n+1} \xi_j \cdot x_j$, then one obtains $n+2$ equations conditioning the constants ξ_j. By rational elimination, it is found that each of the constants $\xi_2, \xi_3, \cdots, \xi_{n+1}$ satisfies an algebraic equation of the nth degree with coefficients which are rational functions of the constants $M_{st} = m_{st} - m_{1t}$ $(s, t = 1, 2, 3, \cdots, n+1; s_1 \neq 1)$. In particular, ξ_{n+1} is a root of the characteristic equation of a certain linear homogeneous substitution.

22. Dr. Alexander's paper contains a simplified proof of Jordan's theorem that a simply closed curve subdivides the plane into two and only two regions. By generalizing the method employed, theorems analogous to Jordan's theorem about k-dimensional manifolds in n-dimensional space are proved.

23. In a paper in the *Transactions* for 1908 Birkhoff considers certain problems connected with the differential system consisting of the equation

$$\frac{d^n u}{dx^n} + * + p_2 \frac{d^{n-2} u}{dx^{n-2}} + \cdots + p_n u + \lambda u = 0$$

and n boundary conditions $W_i(u) = 0$, in which the W's are linear combinations of the values of u and its first $n-1$ derivatives at the end points of the interval over which the system is considered. Among other things he defines a Green's function, $G(x, s, \lambda)$, for this system and proves that for a system with "regular" boundary conditions

$$\lim_{m=\infty} \frac{1}{2\pi i} \int_\Gamma \int_a^b G(x, s, \lambda) f(s) ds d\lambda = f(x),$$

for any function $f(x)$ possessing a continuous derivative, where Γ is a contour in the λ plane enclosing the first m poles of $G(x, s, \lambda)$.

In the present paper Dr. Wilder extends this result in that

he adds to the W's linear combinations of the values of u and its first $n-1$ derivatives at any finite number of points interior to the interval. The Green's function for this system is then defined by the same formula as is used by Birkhoff and it is found that the above integral converges to $f(x)$ provided $f(x)$ has a certain number of derivatives, which number never need exceed n, and provided certain determinants formed from the constants of the auxiliary conditions do not vanish. In the case n is even it is further necessary to assume that the second point from either end of the interval is farther from that end than the first point from the other end is from that end.

24. Professor Huntington's paper refers to the theorem of Duhamel already discussed by Osgood, R. L. Moore, and Bliss in the *Annals of Mathematics* for 1903, 1912, and 1914, namely: If $\alpha_1, \alpha_2, \cdots, \alpha_n$ and $\beta_1, \beta_2, \cdots, \beta_n$ are two sets of infinitesimals such that $\lim_{n=\infty} (\beta_i/\alpha_i) = 1$; and if $\lim_{n=\infty} [\alpha_1 + \alpha_2 + \cdots + \alpha_n] = a$ exists, then $\lim_{n=\infty} [\beta_1 + \beta_2 + \cdots + \beta_n]$ will also exist, and be equal to a. The following example of the failure of this theorem is simpler than examples that have been previously given: Let $\alpha_i = a/n$, and $\beta_i = a/n + 2ic/n^2$, where a and c are fixed constants. Then $\lim \beta_i/\alpha_i = 1$ as $n = \infty$; but $\lim \Sigma\alpha_i = a$, while $\lim \Sigma\beta_i = a + c$.

25. Professor Osgood's paper appeared in full in the June BULLETIN.

26. Professor Wilson's paper appeared in full in the May BULLETIN.

F. N. COLE,
Secretary.

APPLICATION OF AN EQUATION IN VARIABLE DIFFERENCES TO INTEGRAL EQUATIONS.

BY PROFESSOR G. C. EVANS.

(Read before the American Mathematical Society, April 29, 1916.)

IT is known that if the kernel of an integral equation of Volterra type is in the simple form of the difference alone of the

two variables, then the kernel of the resolvent equation is in the same form. We shall see presently that this result also holds for the kernel of the Fredholm equation, provided that it is periodic, the period being equal to the interval of integration. Moreover, the same condition of periodicity is sufficient to make the kernel of the resolvent equation of the same form as the original kernel when the latter is expressed by the formula

$$K(x, y) = \Psi(x + y) + \Theta(x - y). \tag{1}$$

An obvious way to approach the problem is to make use of the partial differential equations

$$\frac{\partial K}{\partial x} + \frac{\partial K}{\partial y} = 0 \quad \text{and} \quad \frac{\partial^2 K}{\partial x^2} = \frac{\partial^2 K}{\partial y^2}, \tag{2}$$

which say respectively that the kernel $K(x, y)$ is a function of the difference of the two variables, or a function of the form (1). The objection to this method is that there is nothing in the theory of the Fredholm equation which demands the existence of derivatives, and nothing which directly refers to them in the statement of the problem.

Less immediate, and also less restrictive, is a treatment by means of Fourier series. The coefficients in the trigonometric development of the resolvent kernel have simple expressions in terms of those of the given kernel, on account of the special form of the latter. One can, in fact, get an instructive aperçu of the facts in the general problem of the Fredholm equation by considering, with these elementary methods, this special case. Here again, however, more seems to be demanded in the nature of the kernel than should be called for by the question which is the subject of this paper.

§ 1. *Some Difference Equations in Variable Differences.*

1. A necessary and sufficient condition that a function $F(x, y)$ be in the form $f(x + y)$ is that it admit the substitution $x' = x + t$, $y' = y - t$, where t is arbitrary. Likewise, a necessary and sufficient condition that a function be in the form $\varphi(x - y)$ is that it admit the substitution $x' = x + t$, $y' = y + t$. These conditions may be respectively expressed by means of the difference equations

$$F(x+t, y-t) - F(x, y) = 0, \tag{3}$$

$$F(x+t, y+t) - F(x, y) = 0. \tag{4}$$

2. In order to distinguish the more interesting case where $K(x, y)$ is of the form (1), let us assume at first that $K(x, y)$ is defined everywhere in the plane except at a point set of measure zero.* *A necessary and sufficient condition that we may write $K(x, y)$ in the form* (1), *except possibly at a set of points of measure zero, is that the condition*

$$K(x+t+t', y+t-t') - K(x+t, y+t) - K(x+t', y-t') + K(x, y) = 0 \tag{5}$$

shall hold for all values of x, y, t, t', except possibly for a point set of zero measure, one of the four points (x, y), $(x+t, y+t)$, $(x+t', y-t')$, $(x+t+t', y+t-t')$ lying in that set.

3. The relation (5) says merely that if we take a rectangle of which the sides are parallel to the lines $x+y=0$ and $x-y=0$, the sum of the values of the function at the extremities of one diagonal is equal to the sum of its values at the extremities of the other, i. e., if 1, 2, 3, 4 are the vertices in cyclic order, we have the equation

$$K_1 + K_3 = K_2 + K_4. \tag{5'}$$

If we have a second rectangle 3, 4, 5, 6 which has a side in common with the first, it is seen that *in the relations*

$$\begin{aligned} K_1 + K_3 &= K_2 + K_4, \\ K_3 + K_5 &= K_4 + K_6, \\ K_1 + K_5 &= K_2 + K_6, \end{aligned} \tag{6}$$

the first two imply the third.

Let us define a null set L as made up of the following one-dimensional sets. Let it include all the lines $x+y=$ const. or $x-y=$ const. on which there is a not-null set of points where $K(x, y)$ is not defined, and let x_0, y_0 be a point where

* The discussion of this problem in connection with equation (5), below, as applied to point sets in general, offers an interesting generalization of some aspects of the theory of the hyperbolic differential equation: in particular, direct proofs of some of the existence theorems. (See a paper by the author soon to be published.

$K(x, y)$ still remains defined. On the line $x + y = x_0 + y_0$ there lie at most a null set of points where $K(x, y)$ is not defined. Include in L the lines $x - y = \text{const.}$ which go through these points; and operate similarly on the line $x - y = x_0 - y_0$. The total set formed in this way constitutes L.

The theorem of Section 2 will be established if we show that *a necessary and sufficient condition that* $K(x, y)$ *be in the form* (1), *except in* L, *is that the relation* (5) *hold unless one of the four points mentioned lie in* L.

4. That the condition is necessary follows at once by direct substitution. In order to show that it is also sufficient, let us write $K(x, y)$ in the form $\Phi(x + y, x - y)$, and consider (5) in the form

$$\Phi(x + y + 2t, x - y + 2t') - \Phi(x + y + 2t, x - y) = \Phi(x + y, x - y + 2t') - \Phi(x + y, x - y),$$

which tells us that the right-hand member is invariant of the substitution $x' = x + t$, $y' = y + t$. We can give t then such a value that $x + y + 2t = x_0 + y_0$, and write

$$\Phi(x_0 + y_0, x - y + 2t') - \Phi(x_0 + y_0, x - y) = \Phi(x + y, x - y + 2t') - \Phi(x + y, x - y)$$

or

$$\Phi(x + y, x - y + 2t') - \Phi(x_0 + y_0, x - y + 2t') = \Phi(x + y, x - y) - \Phi(x_0 + y_0, x - y).$$

The right-hand member of this equation is thus seen to be invariant of the substitution $x' = x + t'$, $y' = y - t'$, wherever it is defined, and so we can give to t' a value such that $x - y + 2t' = x_0 - y_0$. Hence we have

$$\Phi(x + y, x_0 - y_0) - \Phi(x_0 + y_0, x_0 - y_0) = \Phi(x + y, x - y) - \Phi(x_0 + y_0, x - y)$$

or

$$(7)\quad K(x, y) = \Phi(x + y, x_0 - y_0) + \Phi(x_0 + y_0, x - y) - \Phi(x_0 + y_0, x_0 - y_0)$$

which proves the theorem. This result can also be obtained by a more geometrical treatment.

5. If instead of being defined over the entire plane, the function $K(x, y)$ is defined over the rectangle R: $a < x < A$, $b < y < B$, it may be extended over the entire plane by first defining it in the interior of the circumscribed rectangle with sides parallel to the lines $x + y = 0$ and $x - y = 0$, and then defining it as having the value zero on the boundary of this rectangle and over the rest of the plane. The definition in this circumscribed rectangle is established by means of the relation (5′), and its uniqueness follows almost immediately by means of the relation (6). By then applying the theorem of Section 2 we have the following

COROLLARY 1. *The theorem of Section 2 holds, if instead of being defined over the entire plane, except for a point set of measure zero, it is defined, with the same exception, over the rectangle* R: $a < x < A$, $b < y < B$.

We have also the following theorems:

COROLLARY 2. *The condition* (5) *will still be necessary and sufficient if we add the restriction that* $|t|$ *and* $|t'|$ *are to be less than* η, *where* η *is assigned arbitrarily, positive, dependent on* x, y *if desired.*

This theorem is deduced immediately with the help of equation (6).

COROLLARY 3. *If the function* $K(x, y)$ *is continuous at the points in which it is defined, the condition* (5) *may be replaced by one in which* $t' = t$, *i. e., by the condition*

$$K(x + 2t, y) - K(x + t, y + t) - K(x + t, y - t) + K(x, y) = 0. \tag{8}$$

In fact, from (8) it follows that

$$K(x + mt + nt, y + mt - nt) - K(x + mt, y + mt) - K(x + nt, y - nt) + K(x, y) = 0,$$

where m and n are integers, and since every point in R is a point of the form $(x + mt + nt, y + mt - nt)$ or a limiting point of such points, we have the relation (5).

COROLLARY 4. *The relation* (5) *may be replaced by the one obtained from it by interchanging* t *and* t', *or by both relations together, and the theorem will still hold.*

COROLLARY 5. *A necessary and sufficient condition that we may write* $K(x, y)$ *in the form*

$$K(x, y) = f(x) + g(y) \tag{9}$$

is that

$$K(x+t, y+t') - K(x, y+t') - K(x+t, y) + K(x, y) = 0. \tag{9'}$$

§ 2. *Periodicity.*

6. The kernel of an integral equation and the kernel of the resolvent equation are connected by Volterra's relation. If we are dealing with the equation of Fredholm type containing a parameter λ, the relation has the well known form

$$K(x, y) + k_\lambda(x, y) = \lambda \int_a^b K(x, \xi) k_\lambda(\xi, y) d\xi = \lambda \int_a^b k_\lambda(x, \xi) K(\xi, y) d\xi. \tag{10}$$

For generality the integral may be taken in the Lebesgue sense. From (10) it follows obviously that if $K(x, y)$ is periodic in x with a certain period, $k_\lambda(x, y)$ is periodic in x with the same period; and similarly for y.

In regard to the kernels of the form (1), in which we are more specially interested, *a necessary and sufficient condition that* $K(x, y)$ *be periodic, with period* p, *in both arguments, is that* $\Psi(x+y)$ *and* $\Theta(x-y)$, *as functions of a single variable, be each periodic with the same period* p. Obviously the condition is sufficient. To show that it is necessary, notice that we have

$$K(x+t, y+t) - K(x, y) = \Psi(x+y+2t) - \Psi(x+y),$$

whence, taking $t = p$,

$$\Psi(x+y+2p) = \Psi(x+y). \tag{11}$$

But also

$$K(x+p+t, y+t) - K(x+p, y) = \Psi(x+y+p+2t) - \Psi(x+y+p),$$

so that

$$\Psi(x+y+p+2t) - \Psi(x+y+p) = \Psi(x+y+2t) - \Psi(x+y),$$

and if $t = \frac{1}{2}p$,

$$\Psi(x + y + 2p) + \Psi(x + y) = 2\Psi(x + y + p).$$

Comparing this with (11), we have

$$\Psi(x + y + p) = \Psi(x + y).$$

From this and the periodicity of $K(x, y)$ it follows that

$$\Theta(x - y + p) = \Theta(x - y).$$

Nothing is changed in these theorems if we except a point set of zero measure (in the case of the second theorem, a point set of the type L).

§ 3. *Kernels of the Form* $\Psi(x + y) + \Theta(x - y)$.

7. Let us understand the integrals to be taken in the Lebesgue sense, and the function $K(x, y)$ to be summable, with its square, over any two-dimensional region. The Volterra relations (10) hold with this assumption except at a point set of measure zero, and by giving a complete definition of the function $K(x, y)$ they can be made to define a function $k(x, y)$ at every point, finite or infinite, as the case may be. We have the following theorem:

THEOREM 1. *If, except at the points of a set L (see Section 3) of measure zero, the kernel $K(x, y)$ can be written in the form* (1), *in which the functions Ψ and Θ, as functions of a single argument, are periodic with period $b - a$, then with the same exception, the resolvent kernel $k(x, y)$ can be written in the same form, and has the same properties of periodicity.*

As has already been noticed in Section 6, it is the same thing as above to assume that $K(x, y)$ is periodic in each argument with period $b - a$, and the periodicity of $k(x, y)$ follows from it. By virtue of the assumption of periodicity in regard to $K(x, y)$ the point set L is itself periodic.

8. In order to prove the theorem let us perform on the second of the equations (10) the operations indicated in (5), and also the operation obtained from this by interchanging t and t'. Let us denote the result of performing these operations on a function $f(x, y)$ by $H_f(t, t'|x, y)$ and $H_f(t', t|x, y)$ respectively. In forming the resulting pair of equations we have to perform such reductions as the following:

$$\int_a^b k_\lambda(x+t+t', \xi)\Psi(\xi+y+t-t')d\xi$$
$$= \int_{a+t-t'}^{b+t-t'} k_\lambda(x+t+t', \xi-t+t')\Psi(\xi+y)d\xi,$$

which reduces to

$$\int_a^b k_\lambda(x+t+t', \xi-t+t')\Psi(\xi+y)d\xi$$

on account of the periodicity.* Hence we obtain finally the pair of simultaneous integral equations:

$$H_K(t, t'|x, y) + H_k(t, t'|x, y)$$
$$= \lambda\left[\int_a^b H_k(t', t|x, \xi)\Psi(\xi+y)d\xi\right.$$
$$\left. + \int_a^b H_k(t, t'|x, \xi)\Theta(\xi-y)d\xi\right],$$

(12)

$$H_K(t', t|x, y) + H_k(t', t|x, y)$$
$$= \lambda\left[\int_a^b H_k(t', t|x, \xi)\Theta(\xi-y)d\xi\right.$$
$$\left. + \int_a^b H_k(t, t'|x, \xi)\Psi(\xi+y)d\xi\right].$$

In this system of equations the kernels are the functions Ψ and Θ, the known functions are the $-H_K$ and the solutions desired are the functions H_k. It is worth noticing that since the kernels are independent of t and t', any of the functions belonging to $K(x, y)$ gives rise to a solution of the homogeneous system of equations corresponding to (12), and vice versa; hence the characteristic values of λ in (12) are merely the roots of $K(x, y)$. In fact if we write $t = t'$ in (12), the pair of equations reduce to a single equation of which the kernel is $K(x, y)$. The solutions of (12) will then be determined unless λ happens to be root of $K(x, y)$.

Since for given values of t and t' the known functions vanish except at the points of a set of zero measure (i. e., except when

* In the treatment by means of the partial differential equation, this change of variable corresponds to an integration by parts.

one of the seven points (x, y), $(x+t, y+t)$, $(x+t, y-t)$, $(x+t', y+t')$, $(x+t', y-t')$, $(x+t+t', y+t-t')$, $(x+t+t', y-t+t')$ lie in L), the solutions will vanish except at a point set of zero measure. Moreover from the formal character of the solution this set will not contain not-null sets of points on any line parallel to the X or Y axes. Hence the right hand members of (12) will vanish identically, and at all points, even at points of L, we shall have the result

$$H_k(t, t'|x, y) = -H_K(t, t'|x, y), \tag{13}$$

which asserts that $H_k(t, t'|x, y)$ and $H_k(t', t|x, y)$ both vanish for all values of x, y, t, t' such that none of the seven points before mentioned lie in L. Hence, by means of Corollary 4 in Section 5, $k(x, y)$ will have the form (1), except for points of L; and the theorem is proved.

9. If instead of being defined over the entire plane, the kernel $K(x, y)$ is defined only for values of x and y between a and b, the theorem may be extended to cover this case. For if the kernel is in the form (1), it is defined thereby over the interior of the circumscribing rectangle mentioned in Section 5. In fact, that is the region of variation for x and y determined by the arguments $x+y$ and $x-y$, on which depend the functions Ψ and Θ. In order to complete the definition of the function over the entire plane in the manner best to fit with Theorem 1, let us cover the plane by repeating the circumscribing rectangle with the attached values of $K(x, y)$, thus making $K(x, y)$ periodic in x and y separately with period $2(b-a)$. By means of Theorem 1, then, we have the following theorem:

THEOREM 2. *If for values of x and y between a and b the kernel $K(x, y)$ can be written in the form* (1), *in which the functions Ψ and Θ, as functions of a single argument, are periodic with period $b-a$, the hypothesis holding except at a point set of zero measure, of the form L (see Section* 3), *then with the same exception, the resolvent kernel $k(x, y)$ can be written in the same form, and has the same properties of periodicity.*

10. The corresponding theorem is true for functions of the form

$$K(x, y) = \Theta(x-y). \tag{14}$$

In fact, if we make use of the equation

$$K(x+t, y+t) - K(x, y) = 0, \tag{15}$$

we can deduce by means of it, in the same way as before we used the relation (5), the following theorem:

THEOREM 3. *Theorems* 1 *and* 2 *hold if instead of considering kernels of the form* (1) *we consider kernels of the form* (14).

The corresponding theorem is not true for kernels which are functions of $x + y$ alone. In regard to functions of the form (14), however, we may go still further, if they are continuous.

THEOREM 4. *If* $K(x, y)$ *is in the form* (14) *and is continuous, a necessary and sufficient condition that* $k(x, y)$ *should be in the same form is that* Θ, *as a function of a single argument, be periodic with period* $b - a$.

It may be noticed that if the kernel is a function of x alone, or is a function of y alone, the resolvent kernel is of the same form; but if the kernel is of the form (9), that is, a function of x alone plus a function of y alone, the resolvent kernel is not necessarily of the same form. In fact, if $f_1(x)$ is the resolvent associated with $f(x)$, and $g_1(y)$ is the resolvent associated with $g(y)$, then the resolvent associated with $f(x) + g(y) - \int_a^b f(\xi)g(\xi)d\xi$ is $f_1(x) + g_1(y) - f_1(x)g_1(y)(b - a)$.*

§ 4. *Applications.*

11. On account of the fact that kernels of the form (1) need not be continuous, they may often be used as approximations for particular kernels which are not themselves in that form. The resolvent kernel will then again under the hypothesis of Theorem 1 be in the form (1), and the method of development in trigonometric functions of period $b - a$ yield immediate results. Indeed, functions of the form (1) are determined if they are known along a line $x + y = c_1$ and a line $x - y = c_2$; a remark which simplifies especially the treatment of the integral equation of Volterra type, since it leaves only the single function φ to be calculated. The subject of approximation has not been much studied in connection with integral equations.

* See Section 12 below, also Evans, "L'algebra delle funzioni permutabili e non permutabili," *Rendiconti del Circolo Matematico di Palermo*, vol. 34, p. 7.

12. If $K_1(x, y)$ and $K_2(x, y)$ are functions of the form (1), then it is easily shown by means of (5) that the integral combination

$$\int_a^b K_1(x, \xi)K_2(\xi, y)d\xi \tag{16}$$

is of the same form, provided that the hypothesis of periodicity (the period being $b - a$) holds for the parts Ψ_1, Θ_1 and Ψ_2, Θ_2 of K_1 and K_2. This is the integral combination which has been so extensively studied by Volterra.

Now it is known that if we are given two functions K_1 and K_2 of x and y, and their respective resolvent kernels k_1 and k_2, there may be built up out of them by means of the combination (16) a new kernel and its resolvent; in fact, the function

$$k_1(x, y) + k_2(x, y) - \int_a^b k_1(x, \xi)k_2(\xi, y)d\xi \tag{17}$$

is resolvent for the function

$$K_2(x, y) + K_1(x, y) - \int_a^b K_2(x, \xi)K_1(\xi, y)d\xi.* \tag{18}$$

We have then the theorem:

If we have $K_1(x, y) = \Psi_1(x + y) + \Theta_1(x - y)$ *and* $K_2(x, y) = \Psi_2(x + y) + \Theta_2(x - y)$, *where* Ψ_1, Θ_1 *and* Ψ_2, Θ_2, *are functions of a single argument, are periodic with period* $b - a$, *and if we denote by* $k_1(x, y)$ *the function resolvent to* $K_1(x, y)$, *and by* $k_2(x, y)$ *the function resolvent to* $K_2(x, y)$, *then the functions given by* (17) *and* (18) *are of the same form, have the same properties of periodicity, and are themselves mutually resolvent kernels.*

RICE INSTITUTE,
April, 1916.

OPERATORS IN VECTOR ANALYSIS.

BY DR. VINCENT C. POOR.

IN a note in the April BULLETIN on "Changing surface to volume integrals," Professor E. B. Wilson asks why my paper in the January BULLETIN was not made shorter by using the Gibbs-Wilson notation. While the brevity and suggestiveness

* See the footnote to Section 10. For purposes of symmetry and convenience of statemen we have taken $\lambda = 1$ and assumed it not to be a root of K_1 or K_2.

of the Gibbs-Wilson notation is admitted, the note is misleading. For had brevity been the chief aim of my paper the notation of Burali-Forti and Marcolongo could have been made to compare very favorably with Professor Wilson's compact reproduction of the formulas in the Gibbs notation. (Compare the analytic statement of the theorems in the two notations.) However, as one of the purposes of my paper was to exhibit the operational feature of the system of Burali-Forti and Marcolongo, obviously the Gibbs-Wilson notation did not lend itself to this end. Moreover, since the notation of Burali-Forti and Marcolongo is not so well known in this country, some explanation seemed to be necessary.

Professor Wilson says in closing: "The use of words like grad, div, rot is hampering: we no longer write Cubus $\overline{m}$ Census $\overline{p}$ 16 rebus aequatur 40 for $x^3 - 8x^2 - 16 = 40$." Everybody admits the last part of this statement. But we still use for particular kinds of functions or operators such symbols as log, sin, cos, etc., arcsin, etc., sinh, cosh, etc. That the use of such "words" as grad, div, rot is hampering, seems to be a matter of opinion, since they may be used interchangeably with other symbols in both notations.

It is unfortunate that Professor Wilson introduced cartesian coordinates into his proof, since a coordinate system has no proper place in vector analysis. But this seems to be characteristic of the Gibbs-Wilson system. In fact Burali-Forti and Marcolongo have pointed out how the dyadics of Gibbs constantly depend on cartesian coordinates,* a non-linear system.

UNIVERSITY OF MICHIGAN.

SHORTER NOTICES.

Grundlehren der Mathematik. Der zweite Band des ersten Teils: *Algebra.* By EUGEN NETTO. Leipzig, Teubner, 1915. xii+232 pp.

THE Grundlehren der Mathematik, für Studierende und Lehrer is a series of four volumes on the elements of mathematics appearing from the press of B. G. Teubner under joint authorship as follows: Part I (two volumes), Die Grundlehren

* Burali-Forti et Marcolongo, Transformations linéaires, 1912, p. 147.

der Arithmetik und Algebra, by E. Netto and C. Färber; Part II (two volumes), Die Grundlehren der Geometrie, by W. Fr. Meyer and H. Thieme. One of the objects of this series is to present the fundamentals of mathematics with particular regard to the influences which a century's advances in the science, especially in the direction of greater accuracy and completeness, have had upon its elementary phases. In addition it is proposed to present in the second volume of each part extensions which will serve as an introduction to enquiries which a deeper understanding of elementary mathematics renders feasible. There is, therefore, an interest attached to the selection of topics which the authors have made, and the volume Algebra, by Dr. Netto, is in this way noteworthy. We find in this algebra nothing about convergency of series. A topic affording choice between two methods of treatment, one involving analytic functions, as for instance the theory of the roots of unity, is consistently developed by the algebraical method. Certain arithmetical topics usually found in algebras, such as permutations and combinations, are also omitted. On the other hand, there is given a rather complete treatment of differentiation of rational functions, including topics in maxima and minima, two proofs of the fundamental theorem of algebra, a chapter on cyclotomy, and a proof of the Abel-Ruffini theorem.

The developments in the text follow an elementary mode, and contain numerous particular examples solved out. In the first chapter determinants are treated as summations, use being made, in determination of the signs of the terms, of -1 to the power $[\alpha_1\alpha_2 \cdots \alpha_n]$, where the latter symbol stands for the number of transpositions from the order $1, 2, \cdots, n$ afforded by the permutation $\alpha_1, \alpha_2, \cdots, \alpha_n$. With the aid of the properties of this symbol the standard theorems are developed as theorems on summations. The second chapter, on rational functions, discusses and illustrates definitions, graphical representations, continuity and limits, and derivatives are introduced by means of the expansion of a polynomial $f(x + h)$ in powers of h. Among the examples of maxima and minima of rational functions occurs that of the function $f(x) = (a - x)^\alpha (b - x)^\beta$, $(a < b)$, where α, β are parameters representing only positive integers > 1. The special value of such a problem is that it separates into a definite number of cases (four), according to the possibilities (α even, β even), (α even, β odd), etc.

Such a problem, requiring separation into cases, affords to the student a real illustration of scientific method.

Chapter three on integral functions takes up constructions by Lagrange's method of interpolation, Euclid's algorithm, partial fractions, and reducibility. A trial method of resolving a polynomial with integral coefficients into factors with integral coefficients is next discussed and illustrated. There follow Eisenstein's and Gauss's theorems on this problem, the former finding application in Chapter ten in a proof of the irreducibility of $(x^p - 1)/(x - 1)$ (p a prime number). There is then given a discussion of a domain of rationality and functions in a domain. After a brief discussion in Chapter four of elementary properties of equations, Chapter five gives the theory of linear dependence and the practice of solution, both approximate and exact, of systems of linear equations. The next chapter, on resultants and discriminants, contains a proof that the dialytic eliminant is the resultant, a treatment of subresultants and common roots of two equations, and the construction of discriminants in determinant form and in terms of the roots. The seventh chapter gives the solution, in much completeness, of the general equations of the second, third, and fourth orders, following in the latter cases the method of Euler.

The proofs given in Chapter eight that every algebraical equation has a root are respectively the first proof of Gauss, given in his Helmstädt dissertation in 1799, and a proof published by Cauchy in 1821.

Symmetric and alternating functions are treated in Chapter nine, the roots of unity by number-theoretic considerations in Chapter ten. The eleventh contains the theory of cyclotomy, regular polygons, and the f-nomial periods. Cyclic equations and their solution by radicals, and reciprocal equations form the subjects of the twelfth chapter. The succeeding chapters then lead up to the proof of the Abel-Ruffini theorem, treating substitution groups and their functions, and the solvability of algebraic equations in general. There then follows a chapter on transformation, with definitions and illustrations of invariants and covariants and reduction of the general n-ary quadratic form to a sum of squares, and a final chapter on Sturm's theorem.

Netto's treatment is, as a whole, particularly commendable, containing many a deft touch marking the handiwork of a master of his craft.

The standard of typography is high. A few misprints may be noted however: On page 3, line 7, αn should read α_n; on page 30, line 6, for $y \pm 2kn$ read $y \pm 2k\pi$; on page 59, last line, read a_k for a_*; on page 85 in line 19, $a_1d - a_1d$ should read $ad_1 - a_1d$. The proof of page 106 has not been well read. There occur three notations for the same function on this page, viz., $g(x)$, $f'(x)$, and $f_2(x)$. The order of f is m at the top of the page, is changed to n at the middle and so used in two determinants, there being no comment on the change, and the order m is restored at the bottom of the page. In line 3 from the bottom $(1/n^{n-1})R(f_2, f_1)$ should be $(1/n^{n-2})R(f_2, f_1)$. Moreover it is not good usage, we believe, to begin a sentence with a mathematical symbol instead of a capitalized word, as is done in the theorem given at the top of this page. In line 11 of page 115 the last β in the line is wrong font. On page 120 in line 15, $2\sqrt{(a_1^2 - a_0a_1)/a_0^2}$ should be $2\sqrt{(a_1^2 - a_0a_2)/a_0^2}$. In line 3 from the bottom of page 122 read a_4 for a_6. In line 2 of page 123 read x_3x_4 for x_3x_6. The numbering of the formulas in the region of page 123 is confused. Equation (27) referred to in line 6 of this page does not occur in the chapter. This renders line 16 on page 125 unintelligible although it may be a misprint of "Nun liefert (21) wegen (23)." On page 213 in line 4 read y_2 for y_2'.

In the way of general criticism the reviewer thinks it might be urged that the treatment of invariants in the book is much too brief. Quite probably this subject is to be expounded at greater length in the volumes on geometry. But if it could have been found feasible to introduce the notions of invariancy in connection with the solutions of the equations of orders less than 5, at sufficient length to show, for instance, that the roots of the resolvent cubic of the quartic equation are irrational invariants of index 2, the rôle of invariants in the elements of algebra would have been rendered more evident.

O. E. GLENN.

Solid Geometry. By WILLIAM BETZ, A.M., and HARRISON E. WEBB, A.B. With the editorial cooperation of PERCY F. SMITH. Ginn and Company, 1916. xxii+177 pp. Price $0.75.

ON account of the existence of so many other interesting and important topics in mathematics which can be offered to the

man entering college there is a tendency to drop solid geometry from the curriculum altogether or to relegate it to the high schools. As the primary reason for the study of this subject we may assign the training in space perception and the additional knowledge of the universe which comes to the student; as a secondary, the additional training in logical thinking and expression. Since the former can be realized in a comparatively few lessons and the study of plane geometry *should* suffice for the latter, it is necessary that the colleges which offer this course to Freshmen be able to give an account of the faith which is in them.

It is true that the study of plane geometry is too often a mere memorizing of certain stock propositions rather than a training of the logical faculty, and that many freshmen show a lamentable ignorance of all methods of reasoning including those supposed to be geometrical; but the unbeliever will ask the pertinent question as to whether a really scientific course can be appreciated by the man entering college or whether it is better to postpone further study of deductive geometry until the junior or senior year. Certainly the great majority of texts which flood the market are ill-adapted for a course in proper reasoning. A not over-critical examination of one of the most popular texts in solid geometry shows errors in the statements or proofs of more than one third the theorems. In the case referred to it is probably the result of ignorance; but the authors of a recent text confess that the mass of errors introduced into their treatise is the result of deliberate catering to the infant mind. Unless considerable moral self-restraint is exercised in teaching, the use of such texts with a college class is apt to prejudice the student against mathematics if not against the instructor.

It ought to be possible in America, as it is in some other countries, to put geometry, both plane and solid, into interesting and understandable form without sacrificing logic. For solid geometry, Betz and Webb have done this at least as successfully as in any book in English that has come to the reviewer's attention. It is one of three or four texts which seem possible as a basis for a college course and it seems admirably adapted for high school use. Among the excellent features of the book are: a brief preliminary intuitional introduction to three-dimensional thinking; the grouping together of the fundamental axioms and some of the more intuitionally

obvious propositions as preliminary statements on which to base later demonstrational work; as adequate a treatment of the incommensurable as one could expect in an elementary course; sketches of proofs (such as that in regard to the volume of a cylinder) which might be made rigorous were the proper tools and the time available; a brief introduction to coordinates in space; a careful selection of propositions and exercises, an interesting style, and attractive typography.

While the great majority of the errors that are the bane of the ordinary text in solid geometry are here absent, some have been carried over to furnish targets for the critical mathematician. It may be well to cite here one each of three or four types. (1) In No. 592 concerning polyhedrons it is stated that "the lines of intersection of the bounding planes are called the edges; the points of intersection of the edges, the vertices." However, there may be many lines of intersection of the planes which are not edges and many points of intersection of the edges which are not vertices. (2) In proving (No. 555) that, if one of two parallel lines is perpendicular to a plane, the other is also, it is necessary first to prove that the second line meets the plane. (3) The notion of half-plane must be introduced into the discussion of diedral angles. Planes *will* extend beyond a line (see No. 559) whether we wish them to or not. The treatment of No. 579 needs an entire revision; among other criticisms it may be noted that the distance from a point to a half-plane face of a diedral angle has not been defined and can not be defined in any usable manner. In place of this theorem it would be better to introduce V of page 372 and the notions connected therewith. (4) In proving the theorem in regard to the volume of a triangular prism (No. 695) the bases of the two prisms are made to coincide. The question as to whether the prisms will then be on opposite sides of the coinciding bases or on the same side is one of order. To avoid this dilemma a mid-section parallel to the bases might be introduced. While no adequate treatment of the notion "order" is possible in an elementary text, its discussion in such a problem as this and of the orders of the face and diedral angles of two vertical polyedral angles (No. 811) warrants a much more careful and extended treatment. And are not the words "same order" in No. 870 used in an entirely different sense from that implied in No. 810?

Professor Smith has rendered a distinct service to the

mathematics of the country by his editing of several texts; this new volume should share in the wide recognition of worth accorded the series.

R. G. D. RICHARDSON.

The Calculus. By E. W. DAVIS and W. C. BRENKE. Edited by E. R. HEDRICK. New York, The Macmillan Company, 1913. xx+383+63 pp.

"THIS book attempts to preserve the essential features of the calculus, to give the student a thorough training in mathematical reasoning, to create in him a sure mathematical imagination, and to meet fairly the reasonable demand for enlivening and enriching the subject through applications at the expense of purely formal work that contains no essential principle."

This is the closing sentence of the preface. It sets forth four things that the authors attempted to do in writing the book. Probably every author of a calculus consciously attempts the first two. An examination of the current texts however reveals but little evidence that the last two have received adequate attention, although there is a clearly defined tendency towards a fuller recognition of their importance. While the formal type of calculus is pretty definitely standardized, there is no generally recognized norm for one of the type here under review. Accordingly a book of this kind is more difficult to write, and also more difficult to teach, than one of the former kind.

It is obvious to any one at all familiar with teachers of college mathematics that the genus is made up of two clearly defined species; namely, those who reverence the symbol and those whose main interest is in the thing symbolized. This book is obviously and confessedly not for the former. It makes its appeal to those who want our students of calculus to realize that the subject is not primarily a formal one, but that it is vitally connected with physical phenomena and represents an important and significant intellectual achievement of the race. For example, instead of devoting a large amount of space to a discussion of the artifices for integration, the authors have presented integration as a process of reversal of rates. They have done this admirably and have brought home to the student with clearness and force what the process is and why it is important for him to study it. And that is the

one thing needful. It helps to create in him a sure mathematical imagination—a far more important thing than to make him an expert manipulator of forms. It is impossible to do both in the time given to the subject.

There are many other features of the book that are excellent for the same reason. Prominent among these the presentation of Taylor's theorem may be mentioned. This presentation is unusual and has the great and rare merit of making the student see an important use for the theorem. The excellence of the treatment of relative rate of increase and the compound interest law is equally conspicuous. The treatment of simple and damped harmonic motion, the frequent and happy use of parametric equations, and the simple examples in least squares also merit special mention. There is a brief but adequate treatment of simple differential equations.

The book is not free from faults, although many of these are of a mechanical nature and are not inherent in the text. For example, the page is too condensed; a number of important principles and formulas appear in inconspicuous places and are not properly emphasized; it is difficult to use the book for reference. But not all of the shortcomings are of this nature. There are a number of inaccuracies. Some of the exercises are unsuitable. The treatment of infinite series is inadequate.

To sum up, it seems to the reviewer that the authors have been reasonably successful in their effort to enliven and enrich the subject and to present it in such a way as to help create in the student a sure mathematical imagination. He has used the book in his classes since its first appearance and imagines that his students get from it a clearer notion of the essential features of the calculus than they get from any other text he has used. Many of its faults can be overcome by the teacher, whereas its excellent features cannot so successfully be grafted by him upon a text of the formal type. In the reviewer's opinion it is the best elementary calculus now available for use in American colleges. A judicious combination of the good points of this book, the Osgood, and the Franklin, McNutt and Charles would make an ideal text.

WILLIAM BENJAMIN FITE.

Analytische Geometrie der Ebene. By PAUL CRANTZ. Leipzig, Teubner, 1915. v+93 pp.

THIS little book, labeled "Analytic Geometry," deals exclusively with the straight line and the conic sections and contains no discussion of the general second degree equation. As indicated on the title page, it is intended for self-instruction. It is arranged in a very compact manner and probably contains a maximum of material for a book of its size. The idea of a translation of axes is introduced early and is used consistently in getting the equations of the conic sections with axes parallel to the coordinate axes.

No clear-cut definition of the equation of a locus is given. In developing the equation of a locus the author simply shows that all points on the locus satisfy a certain equation, omitting the converse theorem. The methods used in getting the equation of a straight line seem very awkward and make what is really a very simple theorem appear rather complicated. In introducing each of the conic sections, a method of construction is given. This, as well as the discussion of the locus from its equation, is helpful to the self-teaching student.

R. B. ROBBINS.

Wahrscheinlichkeitsrechnung und ihre Anwendung auf Fehlerausgleichung, Statistik und Lebensversicherung. Von EMANUEL CZUBER. Erster Band, dritte Auflage. Leipzig, Teubner, 1914. xii+462 pages.

THE first volume of the second edition of this work appeared in 1908. It was reviewed by H. B. Phillips in this BULLETIN, volume 20, pages 429–431. In this notice of the first volume of the third edition it will therefore be sufficient to call attention to the nature of the changes made in producing it from the corresponding volume of the second edition.

In the six years intervening between the publication of these two editions of volume I the literature of the calculus of probabilities has been enriched by the appearance of several works of importance. Of these mention may be made of the following: Borel's suggestive and illuminating "Eléments de la Théorie des Probabilités"; Liebmann's German translation of Markoff's treatise in the Russian language; a new edition of Poincaré's lectures on the theory of probability revised by the author himself; the first volume of Bachelier's "Calcul des Probabilités"; Carvallo's "Calcul des Probabilités."

Naturally these works have brought out some new results and have emphasized some fresh points of view, at least in particular parts of the theory. But Czuber has not found it necessary to recast his treatise in order to take account of these. It retains the same general form and arrangement as heretofore. But in many places there are minor alterations and improvements and occasionally a new portion of several pages. In addition there are a few minor rearrangements of old matter. On the whole the work is considerably improved. The printing is well done, no typographical errors of importance having been found. On account of its great importance in the theory of probabilities one desires a more satisfactory account of Stirling's formula for the asymptotic character of the gamma function than that given in § 14.

The following is a list of the principal additions: an elegant section (pages 72–80) on the theory of mean value and various applications of it throughout the book; a discussion (pages 83–89) of the use of continuous variables in the theory of probabilities; derivation of formulas (pages 119–128) for the product of binomial factors; a section (pages 239–249) on "Spielprobleme"; an important chapter (pages 273–286) on continuous probabilities in which are developed the fundamental ideas about continuous probabilities in the sense of Bachelier's use of this term; additional matter (see especially pages 413–423) containing a selection of typical problems illustrating the applications in this direction of the analytical representation of arbitrary distributions.

Besides these larger sections, which may be singled out as distinct additions, there are many of less extent scattered throughout the whole volume and contributing essentially to its improvement. As an example of these one may mention the theorems of Bernoulli and Poisson which are now treated from various points of view and illuminated by various analytical lemmas. Through these several improvements the author has accomplished his purpose "den Inhalt nach manchen Richtungen zu erweitern und zu vertiefern."

R. D. CARMICHAEL.

Annuaire pour l'An 1916 *publié par le Bureau des Longitudes.* Paris, Gauthier-Villars, vi+502 pp., with two appendices.

AN excellent "Notice" by M. G. Bigourdan on the mean barometric pressure and law of the winds in France is the chief

feature of the *Annuaire* for 1916. This title is amply fulfilled in the text of the article with its full explanations and in the numerous diagrams. As with so many French writers, the author does not hesitate to start with a brief but clear explanation of the fundamental principles of meteorology, gradually carrying the reader towards the special line of thought which he is investigating, so that the whole constitutes an elementary treatise on the subject. There is also a sympathetic account of Commandant Guyou by M. Emile Picard. M. Guyou had long been connected with the naval activities of France and more particularly with various problems of navigation. He had taken an active part in the publication of the *Annuaire*.

The body of the publication is kept up to date in the usual manner. A useful novelty, and the only one noticed, is a supplement containing the chief astronomical events for the year 1917 compressed into eighteen pages.

ERNEST W. BROWN.

NOTES.

AT the meeting of the London mathematical society held on April 27 the following papers were read: By H. S. CARSLAW: "The Green's function for the equation $\nabla u^2 + k^2 u = 0$, II"; by S. CHAPMAN: "On the uniformity of gaseous density, according to the kinetic theory"; by J. HODGKINSON: "The nodal points of a plane sextic"; by P. A. MACMAHON: "Some problems of combinatory analysis"; by S. POLLARD: "On the deduction of criteria for the convergence of Fourier's series from Fejér's theorem concerning their summability"; by Mrs. G. C. YOUNG: "On the derivatives of a function"; by W. H. YOUNG: "Note on functions of upper and lower type."

AT the meeting of the Edinburgh mathematical society on May 12 the following papers were read: By S. BRODETSKY: "The linear differential equation of the second order"; by D. M. Y. SOMMERVILLE: "A new nomogram for the cubic equation"; by G. PHILIP: "On a group of parabolas associated with the triangle"; by F. G. TAYLOR: "Birationally related cubics."

THE National bureau of the census has recently published a bulletin on the United States life tables prepared by Professor

J. W. GLOVER, of the University of Michigan. Copies can be obtained gratis from the Bureau.

THE final date for receiving memoirs in competition for the King Gustav V prize (see BULLETIN for October, 1913, page 40) has been extended to October 31, 1917.

A new mathematical periodical, the *Revista de Matematicas*, has been founded at Buenos Aires. It will appear monthly under the editorship of Professor MANUEL GUITARTE.

THE following advanced courses in mathematics are announced for the academic year 1916–1917:

MASSACHUSETTS INSTITUTE OF TECHNOLOGY (first half-year):—By Professor H. W. TYLER: Theory of functions of a complex variable, two hours.—By Professor F. S. WOODS: Advanced calculus and differential equations, four hours.—By Professor F. H. BAILEY: Fourier series, two hours.—By Professor E. B. WILSON: Rigid dynamics and hydrodynamics, two hours.—By Professor C. L. E. MOORE: Analytic mechanics, two hours.—By Dr. J. LIPKA: Mathematical laboratory, two hours.

YALE UNIVERSITY:—By Professor JAMES PIERPONT: Theory of functions of a complex variable, two hours; Modern analytic geometry, two hours; Elliptic functions, two hours.—By Professor E. W. BROWN: Advanced calculus, three hours; Advanced dynamics, two hours; Celestial mechanics, two hours.—By Professor P. F. SMITH: Differential equations, two hours.—By ————————: Advanced algebra, two hours.—By Professor E. J. MILES and Dr. H. F. MACNEISH: Differential geometry, two hours.

MR. J. H. HILL has been appointed professor of mathematics at the Ohio Northern University.

PROFESSOR ELLEN HAYES, of Wellesley College, will retire from active service at the end of the present academic year.

MR. C. H. CURRIER, instructor in mathematics in Brown University, has been promoted to an assistant professorship.

MR. F. S. NOWLAN, of Columbia University, has been appointed instructor in mathematics at the Carnegie School of Technology at Pittsburgh.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BLUTEL (E.). Leçons de mathématiques spéciales à l'usage des candidats à l'Ecole Polytechnique et à l'Ecole Normale Supérieure, etc. Tome 1: Algèbre, ligne droite et plan, trigonométrie, analyse, applications géométriques. Tome 2: Géométrie analytique des courbes et des surfaces. Paris, Hachette, 1914. 8vo. 7+635+4+430 pp.

BRADGON (C.). Four dimensional vistas. New York, A. A. Knopl, 1916. $1.25

CHISINI (O.). See ENRIQUES (F.).

DUMONT (E.). Théorie générale des nombres. Définitions fondamentales. (Collection Scientia.) Paris, Gauthier-Villars, 1915. 8vo. 194 pp. Fr. 2.00

ENRIQUES (F.). Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche, pubblicate per cura dell' O. Chisini. Bologna, Zanichelli, 1915. 8vo. 14+397 pp. L. 12.00

GALDEANO (Z. G. DE). Razonamiento de mi curso elemental de calculo infinitesimal comprendiendo nociones de matemática fisico-quimica. Zaragoza, G. Casanal, 1915. 8vo. 244 pp. 5 Pes.

GONSETH (F.). Etude synthétique et applications de l'apolarité. (Thèse, Zürich.) Genève, A. Kundig, 1915. 8vo. 60 pp.

GRÖNFELDT (S.). Systematisk förteckning öfver G. Mittag-Lefflers matematiska bibliotek. Upsala, 1915. 4to. 6+345 pp. M. 31.50

HADAMARD (J.). Four lectures on mathematics delivered at Columbia University in 1911. (Ernest Kempton Adams Fund Publication No. 5.) New York, Columbia University Press, 1915. 4to. 6+52 pp. $0.75

KUPPIS (J.). Die Beweisführung des grossen Satzes Fermats. Budapest, 1915. 8vo. 15 pp. M. 0.50

LORIA (G.). Guida allo studio della storia delle matematiche. (Manuali Hoepli.) Milano, Hoepli, 1916. 16+228 pp. 12mo. L. 3.00

SIBIRANI (F.). Riassunto-formulario di geometria analitica, algebra, calcolo infinitesimale, calcolo vettoriale e meccanica razionale. Roma, tip. Nazionale, di G. Bertero, 1915. 8vo. 64 pp.

STØRMER (C.). Den tredje Skandinaviske Matematikerkongress i Kristiania, 1913. Kristiania, H. Aschehoug, 1915. 8vo. 175 pp.

TEIXEIRA (F. G.). Sur les problèmes célèbres de la géométrie élémentaire non résolubles avec la règle et le compas. Coïmbre, Imprimérie de l'Université, 1915. 4to. 132 pp.

WAHLUND (A.). Sur quelques propriétés des fonctions entières de genre zéro. Upsala, 1915. 8vo. 75 pp. M. 3.00

II. ELEMENTARY MATHEMATICS.

ALVORD (C. P.) and DAVIS (M. E.). Drill and problem book in arithmetic. Syracuse, N. Y., Iroquois Publishing Company, 1916. 238 pp. $0.45

DAVIS (M. E.). See ALVORD (C. P.).

HUNT (B.). Community arithmetic. New York, American Book Company, 1916. 285 pp. 12mo. $0.60

KARVE (R. D.). Practical geometry. Part 1, for Indian schools. Part 2, for Indian colleges. London, Bell, 1915. Cr. 8vo. 1s.+1s. 6d.

MATHEMATICAL PAPERS for admission to the Royal Military College for the years 1906–15. London, Macmillan, 1916. 6s.

PETERS (J.). Dreistellige Tafeln für logarithmisches und numerisches Rechnen. Berlin, s. d. 8vo. 36 pp. Kartonniert. M. 1.00

REED (H. L.). Plane trigonometry. London, Bell, 1915. Cr. 8vo. 320 pp. 3s. 6d.

SHEPPARD (G. Q.). Final year's work in arithmetic. 2d edition. Pottstown, Pa., Hill School, 1915. 159 pp. $1.00

URAGUCHI (Y.). Handy logarithm tables. Tokyo, Uraguchi, 1915. 7 pp. $0.06

III. APPLIED MATHEMATICS.

BARKHAUSEN (H.). See MAXWELL (J. C.).

BARNES (A. A.). Hydraulic flow reviewed. London, Spon, 1916. 12s. 6d.

BARZIZZA (G. B.). Gnomica: l'orologia solare a tempo vero nella sua moderna applicazione. Milano, Hoepli, 1916. 8+199 pp. L. 2.50

BEAUFILS (H.). See HARANG (F.).

BERGERSEN (H.). See GRAN (A.).

BESTELMEYER (A.). See HANDBUCH.

BOCCARDI (G.). Lezioni di cosmografia. Milano, Hoepli, 1916. 9+233 pp. L. 3.00

BOSS (L.). Preliminary catalogue of 6188 stars for the epoch 1900, including those visible to the naked eye and other well-determined stars. Photo-reprint. Washington, 1915. 4to. 37+345 pp. $4.25

COLOMBI (C.). Potentiels et représentations géométriques de la thermodynamique. Paris, Dunod et Pinat, 1914. 95 pp. Fr. 4.50

DAVAUX (E.). See PERRY (J.).

DECKARD (H. C.). Handbook of practical mathematics. Birmingham, Mich., 1916. 82 pp. 16mo. $2.00

DRURY (F. E.). Geometry of building construction. 2d year course. London, Routledge, 1915. 238 pp. 3s.

EFFEMERIDI astronomiche ad uso dei naviganti per l'anno 1916. Genova, tip. r. Istituto idrografico, 1915. 8vo. 196 pp. L. 1.50

EMDE (F.). See MAXWELL (J. C.).

FABRY (E.). Problèmes de mécanique rationnelle à l'usage des candidats aux certificats de licence et à l'aggrégation. Paris, Hermann, 1915. 8vo. 425 pp. Fr. 12.00

FREUCHEN (P. B.). Termodynamik. Grundstraek af Termodynamikens Historie by de to Hovedsaetningers Betydning. Kjöbenhavn, 1915. 8vo. 152 pp. M. 3.60

GABRIEL (E.). Eléments de topographie et tracé des voies de communication. Paris, Gigord, 1915. 8vo. 652 pp.

GEHRCKE (E.). See HANDBUCH.

GOUARD (E.) et HIERNAUX (G.). Cours élémentaire de mécanique industrielle. Principes généraux, applications, exercices pratiques. Tome 1. 2e édition. Paris, Dunod et Pinat, 1914. 12mo. 386 pp. Cartonné. Fr. 4.50

GRAN (A.) og BERGERSEN (H.). Laerebok i navigation. 3. utgave' omarbeidet av O. Johnsen. Del 1. Kristiania, 1915. Gr. 8vo. 8 +408 pp. Bound. M. 11.50

GRIESBACH (H.). Physikalisch-chemische Propädeutik. Unter besonderer Berücksichtigung der medizinischen Wissenschaften und der historischen und biographischen Angaben. 2te Hälfte. 4te Lieferung: Der Schall als besondere Form der mechanischen Energie; Register. Leipzig, 1915. Gr. 8vo. 37+353-1881 pp. (von Band II). M. 80.00

HALLWACHE (W.). See HANDBUCH.

HANDBUCH der Radiologie. Unter Mitwirkung von A. Bestelmeyer, O. Langevin, H. A. Lorentz, E. Rutherford, herausgegeben von E. Marx (4 Bände). Band 3: Glimmentladung, positive Säule und Lichtelektrizität, von E. Gehrcke, R. Seeliger und W. Hallwache. Leipzig, 1916. Gr. 8vo. 22+618 pp. M. 34.00

HARANG (F.) et BEAUFILS (H.). Notions élémentaires de géométrie descriptive appliquée au dessin. Paris, Dunod et Pinat, 1914. Cartonné. Fr. 2.50

HELDERMAN (W. D.). Thermodynamica van het Weston-Normaalelement. Utrecht, 1915. Gr. 8vo. 10+73 pp. M. 4.00

HENNING (F.). Die Grundlagen, Methoden und Ergebnisse der Temperaturmessung. Braunschweig, 1915. Gr. 8vo. 9+297 pp. M. 9.00

HIERNAUX (G.). See GOUARD (E.).

HIGBIE (H. H.). See TIMBIE (W. H.).

IZART (J.). Agenda mécanique à l'usage des ingénieurs, constructeurs-mécaniciens, industriels, chefs d'ateliers et contremaîtres. Paris, Dunod et Pinat, 1916. Toile souple. Fr. 3.00

JEANS (J. H.). The dynamical theory of gases. 2d edition. Cambridge, University Press, 1916. 6+436 pp. 16s.

JOHNSEN (O.). See GRAN (A.).

KOPFF (M.). Die Bahnen der beweglichen Gestirne im Jahre 1916. Berlin, 1916. Gr. 8vo. 10 pp. M. 0.40

LAMB (H.). Hydrodynamics. 4th edition. Cambridge, University Press, 1916. 16+708 pp. 24s.

LANGEVIN (O.). See HANDBUCH.

LORENTZ (H. A.). See HANDBUCH.

MAILLET (E.). Cours de mécanique. Paris, Hermann, 1915. 8vo. 376 pp. Fr. 10.00

MARTENSEN-LARSEN (H.). Stjernehimlens store Problemer. Kjöbenhavn, 1915. 8vo. 330 pp. M. 6.40

MARTIN (L. A.). Text-book of mechanics. Vol. VI: Thermodynamics. New York, Wiley, 1916. 18+313 pp. Cloth. $1.75

MARX (E.). See HANDBUCH.

MATZDORFF (M.). Berechnung des Mondradius aus Bedeckungen von Sternen erster Grösse während der Jahre 1831–1911. Strassburg, 1914. 4to. 60 pp. M. 2.00

MAXWELL (J. C.). Elektrizität und Magnetismus. Auszüge. Uebersetzt von H. Barkhausen, herausgegeben von F. Emde. Braunschweig, 1915. Gr. 8vo. 32+182 pp. M. 7.00

PARKER (G. W.). Elements of optics for the use of schools and colleges. London and New York, Longmans, 1915. 8vo. 6+122 pp. Cloth. $0.75

PERREGAUX (C.) et WEBER (A.). Le relief en géométrie par les couleurs complémentaires. 50 planches de géométrie et de géométrie descriptive. Bienne, E. Magnon, 1915. Fr. 25.00

PERRY (J.). Mécanique appliquée à l'usage des élevès qui peuvent travailler expérimentalement et faire des exercices numériques et graphiques. Ouvrage traduit sur la 9e édition anglaise par E. Davaux. Avec un appendice sur les "toupies tournantes" du même auteur. Tome 2: Constructions déformables et machines en mouvement. Paris, Hermann, 1915. 8vo. 319 pp. Fr. 8.00

PLANCK (M.). Eight lectures on theoretical physics delivered at Columbia University in 1909. Translated by A. P. Wills. (Ernest Kempton Adams Fund Publication No. 3.) New York, Columbia University Press, 1915. 4to. 10+130 pp. $1.00

ROGERS (G. A.). Handbook of fundamental optics. Kansas City, 1915. 8vo. 98 pp. $1.75

RUTHERFORD (E.). See HANDBUCH.

SCHAU (A.). Statik mit Einschluss der Festigkeitslehre. (Aus Natur und Geisteswelt, No. 497.) Leipzig, Teubner, 1915. 8vo. 4+144 pp. M. 1.25

SEAL (B.). The positive sciences of the ancient Hindus. London, Longmans, 1915. 8+295 pp. 12s. 6d.

SEELIGER (R.). See HANDBUCH.

TIMBIE (W. H.) and HIGBIE (H. H.). Alternating-current electricity and its applications to industry. Second course. (Wiley Technical Series.) New York, Wiley, 1916. 8vo. 10+729 pp. Cloth. $3.00

WALKER (M.). Specification and design of dynamo-electric machinery. New York, 1915. 4to. 19+648 pp. $12.00

WEBER (A.). See PERREGAUX (C.).

WELD (L. D.). Theory of errors and least squares. New York, Macmillan, 1916. 12mo. 190 pp. $1.25

WERELDE (T.). Statical theory of energy and matter. Copenhagen, 1915. 8vo. 156 pp. M. 8.50

WILLS (A. P.). See PLANCK (M.).

WILMOTTE (M.). Cours de mécanique à l'usage des écoles industrielles et professionelles. Paris, Béranger, 1915. Cartonné. Fr. 10.00

TWENTY-FIFTH ANNUAL LIST OF PAPERS

READ BEFORE THE AMERICAN MATHEMATICAL SOCIETY AND SUBSEQUENTLY PUBLISHED, INCLUDING REFERENCES TO THE PLACES OF THEIR PUBLICATION.

ALEXANDER, J. W., II. Functions which map the interior of the unit circle upon simple regions. Read April 24, 1915. *Annals of Mathematics*, ser. 2, vol. 17, No. 1, pp. 12–22; Sept., 1915.

ALTSHILLER, N. On the circles of Apollonius. Read Aug. 3, 1915. *American Mathematical Monthly*, vol. 22, No. 8, pp. 261–263; Oct., 1915: No. 9, pp. 304–305; Nov., 1915.

BARROW, D. F. Oriented circles in space. Read Jan. 2, 1915. *Transactions of the American Mathematical Society*, vol. 16, No. 3, pp. 235–258; July, 1915.

BATEMAN, H. A certain system of linear partial differential equations. Read Feb. 26, 1916. *Bulletin of the American Mathematical Society*, vol. 22, No. 7, pp. 329–335; April, 1916.

BEETLE, R. D. On the complete independence of Schimmack's postulates for the arithmetic mean. Read April 25, 1914. *Mathematische Annalen*, vol. 76, No. 4, pp. 444–446; June, 1915.

—— Congruences associated with a one-parameter family of curves. Read Dec. 31, 1913 and Sept. 8, 1914. *American Journal of Mathematics*, vol. 37, No. 3, pp. 281–308; July, 1915.

BENNETT, A. A. The iteration of functions of one variable. Read Aug. 4, 1915. *Annals of Mathematics*, ser. 2, vol. 17, No. 1, pp. 23–60; Sept. 1915.

BERNSTEIN, B. A. A set of four independent postulates for Boolean algebras. Read Aug. 3, 1915. *Transactions of the American Mathematical Society*, vol. 17, No. 1, pp. 50–52; Jan., 1916.

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